DATA PRODUCT SPECIFICATION FOR SALINITY

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in Cooperation with

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## Document Control Sheet

<table>
<thead>
<tr>
<th>Version</th>
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<td>0-01</td>
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<td>M. Gibney</td>
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Signature Page

This document has been reviewed and approved for release to Configuration Management.

OOI Chief Systems Engineer: ________________________________
Date: 15 Dec 11 ________________________________

This document has been reviewed and meets the needs of the OOI Cyberinfrastructure for the purpose of coding this algorithm.

OOI CI Signing Authority: ________________________________
Date: 12-15-11 ________________________________
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1 Abstract
This document describes the computation used to calculate the OOI Level 2 Salinity core data product, which is calculated using the TEOS-10 equations for practical salinity with data from the conductivity, temperature, and depth (CTD) family of instruments. This document is intended to be used by OOI programmers to construct appropriate processes to create the L2 Salinity product.

2 Introduction

2.1 Author Contact Information
Please contact Lorraine Heilman (lheilman@oceanleadership.org) or the Data Product Specification lead (DPS@lists.oceanobservatories.org) for more information concerning the algorithm and other items in this document.

2.2 Metadata Information

2.2.1 Data Product Name
The OOI Core Data Product Name for this product is
- PRACSAL

The OOI Core Data Product Descriptive Name for this product is
- Practical Salinity

2.2.2 Data Product Abstract (for Metadata)
The OOI Level 2 Salinity core data product is computed using the TEOS-10 equations for practical salinity with data from the conductivity, temperature, and depth (CTD) family of instruments.

2.2.3 Computation Name
Not required for data product algorithms.

2.2.4 Computation Abstract (for Metadata)
This algorithm computes the OOI Level 2 Salinity core data product, which is calculated using the TEOS-10 equations for practical salinity with data from the conductivity, temperature, and depth (CTD) family of instruments.

2.2.5 Instrument-Specific Metadata
See Section 4.4 for instrument-specific metadata that must be included with the output of this algorithm.

2.2.6 Synonyms
Synonyms for this data product are
- Salinity
- Seawater salinity

2.2.7 Similar Data Products
Similar products that this may be confused with are
- Absolute Salinity
- Density Salinity

Additional information regarding the similar products can be found at the TEOS-10 website (http://www.teos-10.org) and references listed in Section 2.5.2.

2.3 Instruments

For information on the instruments from which the L2 PRACSAL core data product inputs are obtained, see the CTD Processing Flow document (DCN 1342-00001). This document contains information on instrument classes and make/models; it also describes the flow of data from the CTDs through all of the relevant QC, calibration, and data product algorithms.

Please see the Instrument Application in the SAF for specifics of instrument locations and platforms.

2.4 Literature and Reference Documents


IOC, et al. (2010). The international thermodynamic equation of seawater-2010: Calculation and use of thermodynamic properties. UNESCO.


2.5 Terminology

2.5.1 Definitions

The following terms are defined here for use throughout this document. Definitions of general OOI terminology are contained in the Level 2 Reference Module in the OOI requirements database (DOORS).

**Absolute Salinity:** Measure of absolute salinity adopted and defined jointly by the Intergovernmental Oceanographic Commission (IOC), International Association for the Physical Sciences of the Oceans (IAPSO) and the Scientific Committee on Oceanic Research (SCOR) in 2010 as part of the new standard for calculating the thermodynamic properties of seawater. Units of Absolute Salinity are g kg⁻¹, the mass fraction of dissolved salts in seawater. Absolute Salinity represents, to the best available accuracy, the mass fraction of dissolved solute in a sample of Standard Seawater of the same density as the observed sample (Pawlowicz, 2010).
**Density Salinity:** Another term for Absolute Salinity

**Practical Salinity:** The measure of salinity defined by the Practical Salinity Scale 1978 (PSS-78). Practical Salinity is a unitless quantity that is approximately equivalent to the mass fraction of dissolved solute in seawater, but is not interchangeable with Absolute Salinity. Practical Salinity is an analogue for conductivity of seawater adjusted for temperature and pressure.

**Salinity:** Can refer to either Practical Salinity or Absolute Salinity and is somewhat ambiguous. Generally, salinity refers to Practical Salinity, the standard in effect from 1980 to 2010.

**Standard Seawater:** Seawater with a specific composition of solutes that can be obtained from a defined physical standard, the IAPSO Standard Seawater. Standard Seawater can be evaporated or diluted with pure water to different salinities.

2.5.2 Acronyms, Abbreviations and Notations

General OOI acronyms, abbreviations and notations are contained in the Level 2 Reference Module in the OOI requirements database (DOORS). The following acronyms and abbreviations are defined here for use throughout this document.

- TEOS-10 Thermodynamic Equations of Seawater – 2010
- GSW  Gibbs seawater or Gibbs function for seawater
- ITS-90  International Temperature Scale of 1990
- ITPS-68 International Practical Temperature Scale of 1968
- PSS-78 Practical Salinity Scale 1978

2.5.3 Variables and Symbols

The following variables and symbols are defined here for use throughout this document.

- $p, \ p$  Sea pressure (in situ)
- $t, \ t$  Temperature (in situ)
- $S_p, SP$  Practical salinity
- $C$  Conductivity

3 Theory

3.1 Description

The practical salinity of seawater is a function of temperature, conductivity, and pressure, which are Level 1 sensor products from the CTD instrument.

Practical salinity, based on the Practical Salinity Scale of 1978 (PSS-78), has been the standard in the ocean science community since the standard was developed. Measurements of salinity are calculated and stored in worldwide databases using the 1978 practical salinity scale. The calculation of practical salinity is based on the UNESCO 1980 Practical Salinity algorithm and the TEOS-10 algorithm for practical salinity (called out in the Implementation section) uses the same mathematics.

3.2 Mathematical Theory

Practical Salinity is based on $R_{15}$, the conductivity of the seawater sample at 15 °C and standard pressure:

$$S_p = a_0 + a_1 R_{15}^{1/2} + a_2 R_{15} + a_3 R_{15}^{3/2} + a_4 R_{15}^2 + a_5 R_{15}^{5/2}$$
Since CTDs measure in situ conductivity and not $R_{15}$, it is necessary to convert from $R_{15}$ to in situ conductivity.

First, conductivity is defined as the electrical conductivity of standard seawater at salinity (S) (PSS-78), temperature (t) (IPTS-68), and pressure (p) in decibars (dbar).

The conductivity ratio ($R$) is defined as the ratio of in-situ conductivity against conductivity at a standard of 35 salinity, 15 °C temperature, 0 dbar pressure:

$$ R = \frac{C(S, t, p)}{C(35, 15, 0)} $$

where $C(S, t, p)$ is the observed quantity, $C(35, 15, 0)$ is calculated to be 42.914.

The conductivity ratio can be factored into three parts

$$ R = \frac{C(S, t, p)}{C(S, 0, 0)} \cdot \frac{C(S, 0, 0)}{C(35, t, 0)} \cdot \frac{C(35, t, 0)}{C(35, 15, 0)} $$

where $R_p$, $R_T$, and $r_t$ are all conductivity ratios.

Rearranging, we define $R_T$ in terms of $r_t$ and $R_p$

$$ R_T = R / (R_p r_t) $$

and define $r_t$ and $R_p$ as

$$ r_t = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 $$

where

- $c_0 = 0.6766097$
- $c_1 = 2.00564 \times 10^{-2}$
- $c_2 = 1.104259 \times 10^{-4}$
- $c_3 = -6.9698 \times 10^{-7}$
- $c_4 = 1.0031 \times 10^{-9}$

and

$$ R_p = 1 + \frac{p(e_1 + e_2 p + e_3 p^2)}{1 + d_1 t + d_2 t^2 + (d_3 + d_4 t) R} $$

where

- $e_1 = 2.070 \times 10^{-5}$
- $e_2 = -6.370 \times 10^{-10}$
- $e_3 = 3.989 \times 10^{-5}$
- $d_1 = 3.426 \times 10^{-2}$
- $d_2 = 4.646 \times 10^{-4}$
- $d_3 = 4.215 \times 10^{-1}$
- $d_4 = -3.107 \times 10^{-3}$

Salinity is then defined in terms of $R_T$ as:

$$ S_p = a_0 + a_1 R_T^{1/2} + a_2 R_T + a_3 R_T^{3/2} + a_4 R_T^2 + a_5 R_T^{5/2} + \Delta S $$

where:

$$ \Delta S = \frac{(t - 15)}{1 + k(t - 15)} (b_0 + b_1 R_T^{1/2} + b_2 R_T + b_3 R_T^{3/2} + b_4 R_T^2 + b_5 R_T^{5/2}) $$

And the constants are defined as

- $a_0 = 0.008$
- $b_0 = 0.0005$
- $a_1 = -0.1692$
- $b_1 = -0.0056$
- $a_2 = 25.3851$
- $b_2 = -0.0066$
- $a_3 = 14.0941$
- $b_3 = -0.0375$
- $a_4 = -7.0261$
- $b_4 = 0.0636$
\( a_5 = 2.7081 \quad b_5 = -0.0144 \)
\( k = 0.0162 \)

for \(-2^\circ C < t < 35^\circ C\) and \(0.2 < S_P < 42\). As noted in Section 3.3 this algorithm is only applicable for these ranges of temperature and salinity.

See Fofonoff and Millard (1983) for further discussion of these algorithms.

### 3.3 Known Theoretical Limitations

The Practical Salinity calculation is valid only in the range \(2 < S_P < 42\) and \(-2^\circ C < \text{temperature} < 35^\circ C\) (Feistel, 2008).

### 3.4 Revision History

No revision to date.

### 4 Implementation

#### 4.1 Overview

All of the functions described here are from the GSW Toolbox provided at the TEOS-10 website (http://www.teos-10.org). As of writing, toolbox GSW 3.0 is available in Matlab, Fortran, and C. Section 4.8 contains more detailed information on these functions. Appendix A contains copies of all of the functions called to document exactly how this function was implemented.

Practical salinity (\(S_P\)) is calculated using the TEOS-10 function

\[
S_P = \text{gsw}_\text{SP}_{\text{from}_C}(C, t, p)
\]

where \(C\) is conductivity (mS/cm), \(t\) is temperature (°C), and \(p\) is pressure (dbar). A simple conversion is required to convert \(C\) from the units of the L1 data product (S/m) to (mS/cm). See section 4.4 for additional information on the inputs.

#### 4.2 Inputs

Inputs

- L1 Temperature [°C] (see 1341-00010_Data_Product_SPEC_TEMPWAT)
- L1 Pressure (sea pressure) [dbar] (see 1341-00020_Data_Product_SPEC_PRESWAT)
- L1 Conductivity [S/m] (see 1341-00030_Data_Product_SPEC_CONDWAT)

All inputs are double precision floating point numbers.

The Practical Salinity calculation is valid only when the input temperature is in the range \(-2^\circ C < t < 35^\circ C\) (Feistel, 2008). This upper range is smaller than the global range check applied to the L1 Temperature input (\(t\)). Because temperatures measured for OOI are never expected to be over 35°C, the upper limit of temperature as an input to this algorithm is not checked.

Input Data Formats

Not applicable for Level 2 Data products.

#### 4.3 Processing Flow

The specific steps necessary to create all calibrated and quality controlled data products for the CTD are described in the CTD Processing Flow document (DCN 1342-00001). This processing flow document contains a flow diagram showing all of the specific algorithms (data products and
QC) necessary to compute all data products from the CTD and the order in which the algorithms must be applied.

The processing flow for this algorithm consists of two steps:
1) Convert the L1 Conductivity input from units \([S/m]\) to units \([mS/cm]\)
   \[ C \ [mS/cm] = \text{L1 Conductivity} \ [S/m] \times 10 \]
2) Call the function \( \text{gsw SP from C}(C,t,p) \) with inputs of Level 1 Temperature \((t, \text{L1 input})\), Pressure \((p, \text{L1 input})\), and the converted conductivity (Step 1 output).

Practical Salinity \((SP)\) is a unitless quantity and should be reported without units.

4.4 Outputs

The outputs of the practical salinity algorithm are
- Practical salinity of seawater (unitless – PSS scale) as a double precision floating point number with six decimal places \%.6f.

Note that practical salinity calculated as described in Section 4.0 Implementation is valid for \(0 < SP < 42\), despite the fact that the PSS-78 calculation is valid only when the output is in the range \(2 < SP < 42\). This is because, if PSS-78 produces a practical salinity below 2, the TEOS-10 Matlab function given will recalculate salinity using a modified form of the Hill et al. (1986) formula. The range check on the output \((0 < SP < 42)\) is performed as part of the global range check on practical salinity (GLBLRNG, DCN 1341-10004) as specified in the CTD Processing Flow document (DCN 1342-00001).

The metadata that must be included with the output of this algorithm are
- the version of the GSW software used (GSW Toolbox 3.0)
- the version of the Matlab execution engine, if used.

See Appendix A for a discussion of the accuracy of the algorithm output.

4.5 Computational and Numerical Considerations

4.5.1 Numerical Programming Considerations
None.

4.5.2 Computational Requirements
Not significant.

4.6 Code Verification and Test Data Sets

The code will be verified using the test data set provided, which contains inputs and their associated correct outputs. CI will verify that the algorithm code is correct by checking that the algorithm density output, generated using the test data inputs, is identical to the test data output and the appropriate metadata have been added to the output.

A test data set below provides a few data points over the extent of the water column. It includes inputs and outputs for the calculation of Practical Salinity.
<table>
<thead>
<tr>
<th>Conductivity [S/m]</th>
<th>Temp (ITS-90) [deg C]</th>
<th>Sea Pressure [dbar]</th>
<th>Practical Salinity (PSS-78) [unitless]</th>
</tr>
</thead>
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<tr>
<td>5.407471</td>
<td>28</td>
<td>0</td>
<td>33.495229</td>
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<tr>
<td>5.407880</td>
<td>28</td>
<td>10</td>
<td>33.495224</td>
</tr>
<tr>
<td>5.041008</td>
<td>20</td>
<td>150</td>
<td>36.995774</td>
</tr>
<tr>
<td>3.463402</td>
<td>6</td>
<td>800</td>
<td>34.898526</td>
</tr>
<tr>
<td>3.272557</td>
<td>3</td>
<td>2500</td>
<td>34.999244</td>
</tr>
<tr>
<td>3.273035</td>
<td>2</td>
<td>5000</td>
<td>34.999494</td>
</tr>
</tbody>
</table>
Appendix A  Example Code

This Appendix contains all Matlab subroutines necessary for calculating Practical Salinity using the GSW toolbox (TEOS-10) available under Creative Commons license from http://www.teos-10.org/. This code has been verified by the originators of the code using examples from the oceanographic community. The current version at the time of writing is GSW Toolbox 3.0.

Matlab, Fortran, and C routines for calculating density are available in the GSW toolbox from the TEOS-10 website.

http://www.teos-10.org/
http://www.teos-10.org/software.htm#1

A copy of the toolbox has been placed on Alfresco.
REFERENCE >> DPS Artifacts >> DENSITY_Code_gsw_matlab_v30_0.zip

A.1  Practical Salinity from conductivity, C

USAGE:    SP = gsw_SP_from_C(C,t,p)

DESCRIPTION: Calculates Practical Salinity, SP, from conductivity, C, primarily using the PSS-78 algorithm. Note that the PSS-78 algorithm for Practical Salinity is only valid in the range 2 < SP < 42. If the PSS-78 algorithm produces a Practical Salinity that is less than 2 then the Practical Salinity is recalculated with a modified form of the Hill et al. (1986) formula. The modification of the Hill et al. (1986) expression is to ensure that it is exactly consistent with PSS-78 at SP = 2. Note that the input values of conductivity need to be in units of mS/cm (not S/m).

INPUT:    C = Conductivity [ mS/cm ]
          t = in-situ temperature (ITS-90) [ deg C ]
          p = sea pressure [ dbar ] ( i.e., absolute pressure - 10.1325 dbar )

          t & p may have dimensions 1x1 or Mx1 or 1xN or MxN, where C is MxN.

OUTPUT:   SP = Practical Salinity on the PSS-78 scale [ unitless ]

AUTHOR:   Trevor McDougall and Paul Barker

VERSION NUMBER:  3.01 (23rd May, 2011)


function SP = gsw_SP_from_C(C,t,p)

% gsw_SP_from_C                        Practical Salinity from conductivity
%=====================================================================% % USAGE: %  SP = gsw_SP_from_C(C,t,p)
% % DESCRIPTION: %  Calculates Practical Salinity, SP, from conductivity, C, primarily using % the PSS-78 algorithm. Note that the PSS-78 algorithm for Practical % Salinity is only valid in the range 2 < SP < 42. If the PSS-78 % algorithm produces a Practical Salinity that is less than 2 then the % Practical Salinity is recalculated with a modified form of the Hill et % al. (1986) formula. The modification of the Hill et al. (1986) % expression is to ensure that it is exactly consistent with PSS-78 % at SP = 2. Note that the input values of conductivity need to be in % units of mS/cm (not S/m).
% % INPUT: %  C  =  conductivity                                             [ mS/cm ] %  t  =  in-situ temperature (ITS-90)                             [ deg C ] %  p  =  sea pressure                                              [ dbar ] %   ( i.e. absolute pressure - 10.1325 dbar ) %  t & p may have dimensions 1x1 or Mx1 or 1xN or MxN, where C is MxN. % % OUTPUT: %  SP  =  Practical Salinity on the PSS-78 scale               [ unitless ] % % AUTHOR: %  Paul Barker, Trevor McDougall and Rich Pawlowicz   [ help_gsw@csiro.au ] % % VERSION NUMBER: 3.0 (1st April, 2010)
% % REFERENCES:
%  Culkin and Smith, 1980: Determination of the Concentration of Potassium %  Chloride Solution Having the Same Electrical Conductivity, at 15C and %  Infinite Frequency, as Standard Seawater of Salinity 35.0000 %  (Chlorinity 19.37394), IEEE J. Oceanic Eng, 5, 22-23.
% The software is available from http://www.TEOS-10.org
%=====================================================================% % Check variables and resize if necessary
% if ~(nargin == 3)  
  error('gsw_SP_from_C: Requires three input arguments')  
end %if

[mc, nc] = size(C);  
[mt, nt] = size(t);  
[mp, np] = size(p);

if (mt == 1) & (nt == 1)  
  % t scalar - fill to size of C  
  t = t*ones(size(C));  
elseif (nc == nt) & (mt == 1)  
  % t is row vector,  
  t = t(ones(1,mc), :);  
elseif (mc == mt) & (nt == 1)  
  % t is column vector,  
  t = t(:,ones(1,nc));  
elseif (nc == mt) & (nt == 1)  
  % t is a transposed row vector,  
  t = t.';  
  t = t(ones(1,mc), :);  
elseif (mc == mt) & (nc == nt)  
  % ok  
else  
  error('gsw_SP_from_C: Inputs array dimensions arguments do not agree')  
end %if

if (mp == 1) & (np == 1)  
  % p scalar - fill to size of C  
  p = p*ones(size(C));  
elseif (nc == np) & (mp == 1)  
  % p is row vector,  
  p = p(ones(1,mc), :);  
elseif (mc == mp) & (np == 1)  
  % p is column vector,  
  p = p(:,ones(1,nc));  
elseif (nc == mp) & (np == 1)  
  % p is a transposed row vector,  
  p = p.';  
  p = p(ones(1,mc), :);  
elseif (mc == mp) & (nc == np)  
  % ok  
else  
  error('gsw_SP_from_C: Inputs array dimensions arguments do not agree')  
end %if

if mc == 1  
  C = C.';  
  t = t.';  
  p = p.';  
  transposed = 1;  
else  
  transposed = 0;  
end

%--------------------------------------------------------------------------------------
% Start of the calculation  
%--------------------------------------------------------------------------------------

a0 = 0.0080;  
a1 = -0.1692;  
a2 = 25.3851;  
a3 = 14.0941;  
a4 = -7.0261;  
a5 = 2.7081;
b0 = 0.0005;
b1 = -0.0056;
b2 = -0.0066;
b3 = -0.0375;
b4 = 0.0636;
b5 = -0.0144;

c0 = 0.6766097;
c1 = 2.00564e-2;
c2 = 1.104259e-4;
c3 = -6.9698e-7;
c4 = 1.0031e-9;

d1 = 3.426e-2;
d2 = 4.464e-4;
d3 = 4.215e-1;
d4 = -3.107e-3;

e1 = 2.070e-5;
e2 = -6.370e-10;
e3 = 3.989e-15;

k = 0.0162;

\[ t_{68} = t \times 1.00024; \]
\[ f_{t_{68}} = \frac{(t_{68} - 15)}{(1 + k \times (t_{68} - 15))}; \]

% The dimensionless conductivity ratio, R, is the conductivity input, C, % divided by the present estimate of C(SP=35, t_68=15, p=0) which is % 42.9140 mS/cm (=4.29140 S/m), (Culkin and Smith, 1980).
\[ R = \frac{0.023302418791070513 \times C}{42.9140}; \]

% rt_lc corresponds to rt as defined in the UNESCO 44 (1983) routines.
rt_lc = c0 + (c1 + (c2 + (c3 + c4 * t_{68})).*t_{68}).*t_{68}.*t_{68};

Rp = 1 + (p.*{(e1 + e2.*p + e3.*p.*p)}. / ...
(1 + d1.*t_{68} + d2.*t_{68}.*t_{68} + (d3 + d4.*t_{68})).*R);

Rt = R./(Rp.*rt_lc);

[Ierror] = find(Rt < 0);
if ~isempty(Ierror)
    Rt(Ierror) = NaN;
end

Rtx = sqrt(Rt);

SP = a0 + (a1 + (a2 + (a3 + (a4 + a5.*Rtx)).*Rtx).*Rtx).*Rtx + ...  
   f_{t_{68}}.*{(b0 + (b1 + (b2 + (b3 + (b4 + b5.*Rtx)).*Rtx).*Rtx).*Rtx}).*Rtx).*Rtx);

% The following section of the code is designed for SP < 2 based on the % Hill et al. (1986) algorithm. This algorithm is adjusted so that it is % exactly equal to the PSS-78 algorithm at SP = 2.
[I2] = find(SP < 2);
if ~isempty(I2)
    Hill_ratio = gsw_Hill_ratio_at_SP2(t(I2));
    x = 400*Hill_ratio(I2);
    sqrty = 10*Rtx(I2);
    part1 = 1 + x.*(1.5 + x);
    part2 = 1 + sqrty.*(1 + sqrty.*(1 + sqrty));
end
SP_Hill_raw = SP(I2) - a0./part1 - b0.*ft68(I2)./part2;
SP(I2) = Hill_ratio.*SP_Hill_raw;
end

% These few lines ensure that SP is non-negative.
[I_neg_SP] = find(SP < 0);
if ~isempty(I_neg_SP)
    SP(I_neg_SP) = 0;
end

if transposed
    SP = SP.';
end
end
A.2 Hill ratio at a Practical Salinity of 2

function Hill_ratio = gsw_Hill_ratio_at_SP2(t)

% gsw_Hill_ratio_at_SP2                                  Hill ratio at SP of 2
%======================================================================================================%
% USAGE:                                                %
%  Hill_ratio = gsw_Hill_ratio_at_SP2(t)                %
% DESCRIPTION:                                        %
%  Calculates the Hill ratio, which is the adjustment needed to apply for %
%  Practical Salinities smaller than 2. This ratio is defined at a %
%  Practical Salinity = 2 and in-situ temperature, t using PSS-78. The Hill %
%  ratio is the ratio of 2 to the output of the Hill et al. (1986) formula %
%  for Practical Salinity at the conductivity ratio, Rt, at which Practical %
%  Salinity on the PSS-78 scale is exactly 2.            %
% INPUT:                                               %
%  t = in-situ temperature (ITS-90)                     [ deg C ]
% OUTPUT:                                              %
%  Hill_ratio = Hill ratio at SP of 2                   [ unitless ]
% AUTHOR:                                              %
%  Trevor McDougall and Paul Barker                   [ help_gsw@csiro.au ]
% VERSION NUMBER: 3.0 (26th March, 2011)                %
% REFERENCES:                                         %
%  Hill, K.D., T.M. Dauphinee & D.J. Woods, 1986: The extension of the %
%  Practical Salinity Scale 1978 to low salinities. IEEE J. Oceanic Eng., %
%  11, 109 - 112.                                      %
% IOC, SCOR and IAPSO, 2010: The international thermodynamic equation of %
% seawater - 2010: Calculation and use of thermodynamic properties. %
% Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, %
% UNESCO (English), 196 pp. Available from http://www.TEOS-10.org %
% See appendix E of this TEOS-10 Manual.
% Unesco, 1983: Algorithms for computation of fundamental properties of %
% The software is available from http://www.TEOS-10.org %
%======================================================================================================%

% Check variables                                        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if ~(nargin == 1)
    error('gsw_Hill_ratio_at_SP2: Needs only one input argument')
end %if
SP2 = 2.*(ones(size(t)));%
%-------------------------------
% Start of the calculation
%-------------------------------
a0 =  0.0080;
a1 =  -0.1692;
a2 =  25.3851;
a3 =  14.0941;
a4 =  -7.0261;
a5 =  2.7081;
b0 =  0.0005;
b1 =  -0.0056;
b2 =  -0.0066;
b3 =  -0.0375;
b4 =  0.0636;
b5 =  -0.0144;
g0 = 2.641463563366498e-1;
g1 = 2.007883247811176e-4;
g2 = 4.107694432853053e-6;
g3 = 8.401670882091225e-8;
g4 = -1.711392021989210e-9;
g5 = 3.37419389377380e-11;
g6 = -5.923731174730784e-13;
g7 = 8.057771569962299e-15;
g8 = -7.054313817447962e-17;
g9 = 2.85999271347235e-19;
k  = 0.0162;
t68 = t.*1.00024;
ft68 = (t68 - 15)./(1 + k.*(t68 - 15));%
%-------------------------------
% Find the initial estimates of Rtx (Rtx0) and of the derivative dSP_dRtx
% at SP = 2.
%-------------------------------
Rtx0 = g0 + t68.*(g1 + t68.*(g2 + t68.*(g3 + t68.*(g4 + t68.*(g5...
       + t68.*(g6 + t68.*(g7 + t68.*(g8 + t68.*g9))))))));
dSP_dRtx =  a1 + (2*a2 + (3*a3 + (4*a4 + 5*a5.*Rtx0).*Rtx0).*Rtx0).*Rtx0 + ...% 
ft68.*(b1 + (2*b2 + (3*b3 + (4*b4 + 5*b5.*Rtx0).*Rtx0).*Rtx0).*Rtx0).*Rtx0);%
%-------------------------------
% Begin a single modified Newton-Raphson iteration to find Rt at SP = 2.
%-------------------------------
SP_est = a0 + (a1 + (a2 + (a3 + (a4 + a5.*Rtx).Rtx0).Rtx0).Rtx0).Rtx0 + ...
       + ft68.*(b0 + (b1 + (b2+ (b3 + (b4 + b5.*Rtx).*Rtx0).*Rtx0).*Rtx0).*Rtx0).*Rtx0;Rtx = Rtx0 - (SP_est - SP2)/dSP_dRtx;
Rtxm = 0.5*(Rtx + Rtx0);
dSP_dRtx =  a1 + (2*a2 + (3*a3 + (4*a4 + 5*a5.*Rtxm).*Rtxm).Rtxm).Rtxm + ...% 
ft68.*(b1 + (2*b2 + (3*b3 + (4*b4 + 5*b5.*Rtxm).*Rtxm).*Rtxm).*Rtxm).*Rtxm;Rtx = Rtx0 - (SP_est - SP2)/dSP_dRtx;
% This is the end of one full iteration of the modified Newton-Raphson
% iterative equation solver. The error in Rtx at this point is equivalent
% to an error in SP of 9e-16 psu.

x = 400*Rtx.*Rtx;
sqrty = 10*Rtx;
part1 = 1 + x.*(1.5 + x);
part2 = 1 + sqrty.*(1 + sqrty.*(1 + sqrty));
SP_Hill_raw_at_SP2 = SP2 - a0./part1 - b0.*ft68./part2;

Hill_ratio = 2./SP_Hill_raw_at_SP2;

end
Appendix B  Output Accuracy
The accuracy of the salinity calculated by this algorithm is a function of the accuracy of the algorithm inputs: pressure, temperature, and conductivity. Using the accuracies specified in the OOI requirements database (DOORS) for the algorithm inputs:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductivity above 200 m</td>
<td>±0.003 mS/cm</td>
</tr>
<tr>
<td>Conductivity below 200 m</td>
<td>±0.01 mS/cm</td>
</tr>
<tr>
<td>Temperature</td>
<td>±0.002 °C</td>
</tr>
<tr>
<td>Pressure</td>
<td>0.1% of full-scale value</td>
</tr>
</tbody>
</table>

and assuming typical ocean values, 5 deg C and 29 mS/cm conductivity, the resulting practical salinity inaccuracy is 0.01 at 100m (mainly from conductivity) and 0.005 at 5000m (from conductivity and pressure) using the algorithm code described in section 4.8.

The following requirements in the DOORS database describe salinity accuracy requirements and are consistent with these calculations:

L2-SR-RQ-3684  Salinity shall be calculated to an accuracy of 0.005 on the PSS, using laboratory measurements of conductivity, temperature, and pressure.

L2-SR-RQ-3833  Salinity shall be calculated to an accuracy of ±0.005 on the PSS using measurements of conductivity, temperature, and pressure taken at depths greater than 200m.

L2-SR-RQ-3834  Salinity shall be calculated to an accuracy of ±0.01 on the PSS using measurements of conductivity, temperature, and pressure taken at depths less than or equal to 200m.
Appendix C  Sensor Calibration Effects

None.